Geometry: 4.1-4.6 Notes

NAME______________________

4.1 Be able to perform translations___________________________________ Date:________________

Define Vocabulary:

vector – A quantity that has both direction and magnitude and is represented in the coordinate plane by an arrow drawn from one point to another

initial point – The starting point of a vector

terminal point – The ending point of a vector

horizontal component – The horizontal change from the starting point of a vector to the ending point

vertical component – The vertical change from the starting point of a vector to the ending point

component form – A form of a vector that combines the horizontal and vertical components

transformation – A function that moves or changes a figure in some way to produce a new figure

image – A figure that results from the transformation of a geometric figure

preimage – The original figure before a transformation

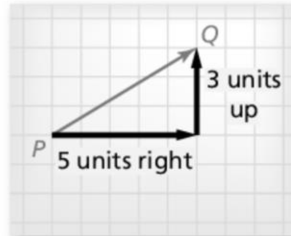
translation – A transformation that moves every point of a figure the same distance in the same direction

rigid motion – A transformation that preserves length and angle measure

composition of transformations – The combination of two or more transformations to form a single transformation

Vectors

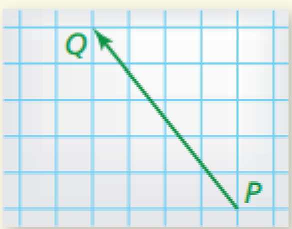
The diagram shows a vector. The **initial point**, or starting point, of the vector is P, and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The **horizontal component** of $\overrightarrow{PQ}$ is 5, and the **vertical component** is 3. The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3 \rangle$.



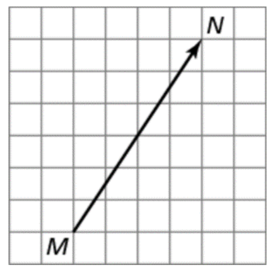
Examples: Identify vector components.

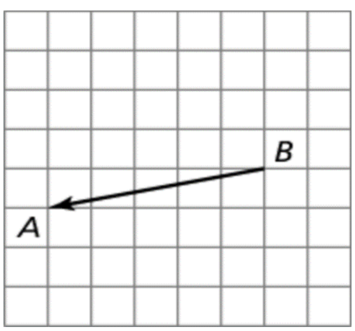
<u>WE DO</u>

Name the vector and write its component form.



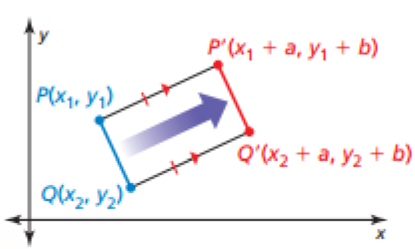
<u>YOU DO</u>





Translations

A **translation** moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves, the points P and Q of a plane figure along a vector $\langle a, b \rangle$ to the points P' and Q', so that one of the following statements is true.

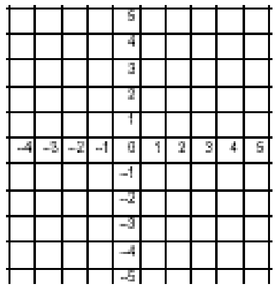


- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

Examples: Translating a figure using a vector.

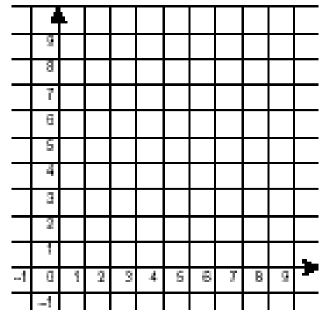
WE DO

The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle -1, -2 \rangle$.



YOU DO

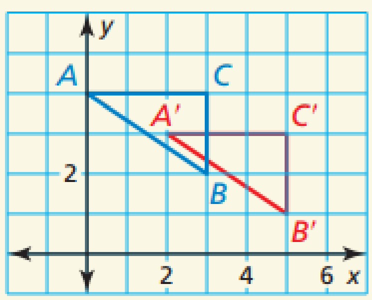
The vertices of $\triangle LMN$ are $L(2, 2)$, $M(5, 3)$, and $N(9, 1)$. Translate $\triangle LMN$ using the vector $\langle -2, 6 \rangle$.



Examples: Writing a translation rule.

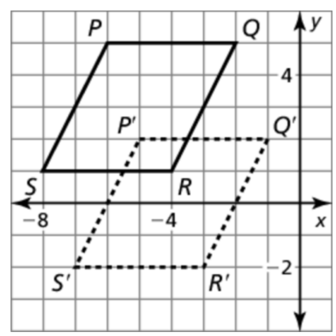
WE DO

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$.



YOU DO

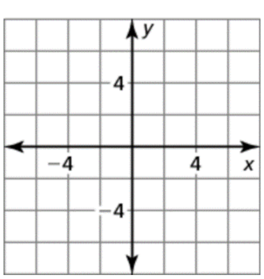
Write a translation rule.



Examples: Translating a figure in a coordinate plane.

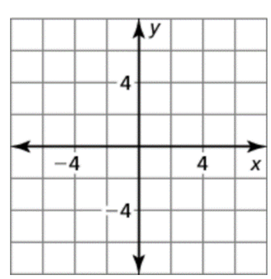
WE DO

Graph quadrilateral $ABCD$ with vertices $A(1, -2)$, $B(2, 1)$, $C(4, 1)$, and $D(4, -2)$ and its image after the translation $(x, y) \rightarrow (x - 1, y + 4)$.



YOU DO

Graph ΔRST with vertices $R(2, 2)$, $S(5, 2)$, and $T(3, 5)$ and its image after the translation $(x, y) \rightarrow (x + 1, y + 2)$.



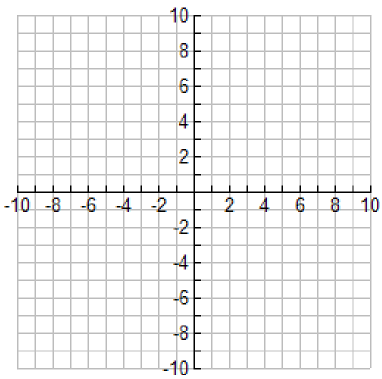
Examples: Performing a composition.

<u>WE DO</u>

Graph $\overline{RS}$ with endpoints $R(-8, 5)$ and $S(-6, 8)$. Graph its image after the composition.

Translation: $(x, y) \rightarrow (x - 1, y + 4)$

Translation: $(x, y) \rightarrow (x + 4, y - 6)$

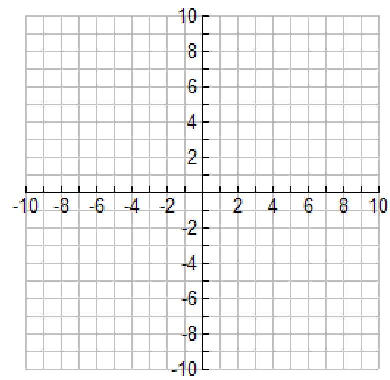


<u>YOU DO</u>

Graph $\overline{TU}$ with endpoints $T(1, 2)$ and $U(4, 6)$ and its image after the composition.

Translation: $(x, y) \rightarrow (x - 2, y - 3)$

Translation: $(x, y) \rightarrow (x - 4, y + 5)$



Assignment	

4.2 Be able to perform reflections.____ Date:_____________

Define Vocabulary:

reflection –

line of reflection –

glide reflection –

line symmetry –

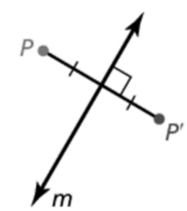
line of symmetry –

Reflections

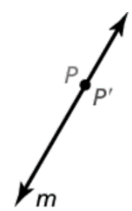
A **reflection** is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P', so that for each point on of the following properties is true.

- If P is not m, then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m, then $P = P'$.



point P not on m



point P on m

Coordinate Rules for Reflections

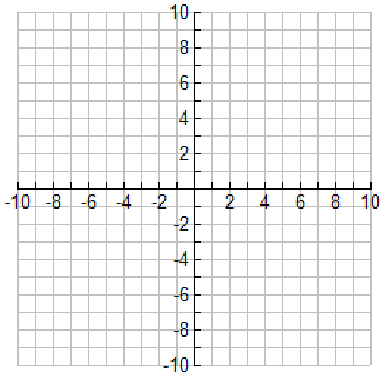
- If (a, b) is reflected in the x-axis, then its image is the point $(a, -b)$.
- If (a, b) is reflected in the y-axis, then its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, then its image is the point (b, a).
- If (a, b) is reflected in the line $y = -x$, then its image is the point $(-b, -a)$.

Examples: Reflecting across horizontal and vertical lines.

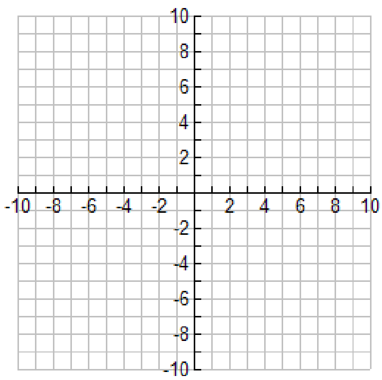
Graph triangle ABC with the vertices A(1, 3), B(5, 2), and C(2, 1) and its image after the reflection described.

<u>WE DO</u>

x = -1

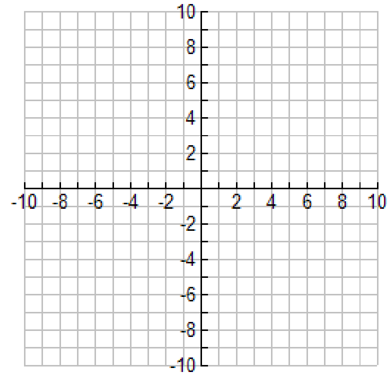


y = 3

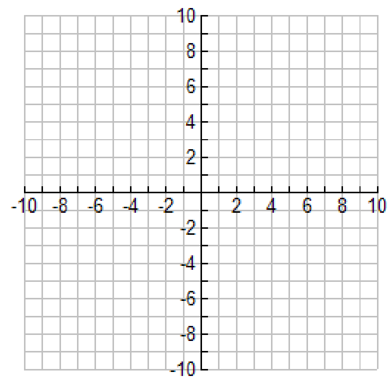


<u>YOU DO</u>

x = 4



y = -2

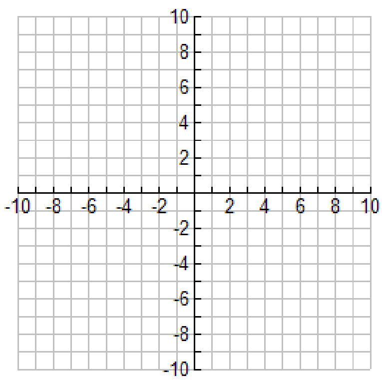


Examples: Reflections

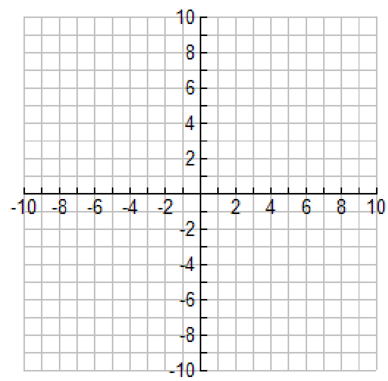
<u>WE DO</u>

Graph $\overline{AB}$ with the endpoints A(3, -1) and B(3, 2) and its image after the described reflections.

1.) ***y = x***



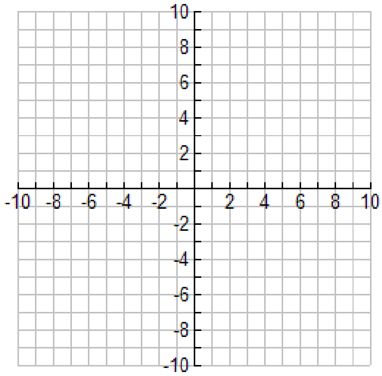
2.) x-axis



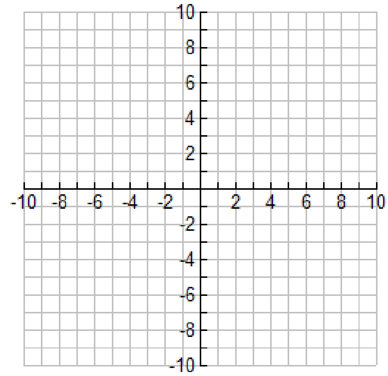
YOU DO

Graph triangle JKL with the vertices J(1, 3) and K(4, 4), and L(3, 1) and its image after the described reflections.

1.) $y = -x$



2.) y -axis

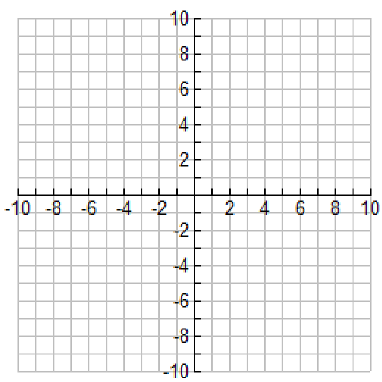


Examples: Graph triangle ABC with the vertices A(3, 2) and B(6, 3), and C(7, 1) and its image after the glide reflection.

WE DO

Translation: $(x, y) \rightarrow (x, y - 6)$

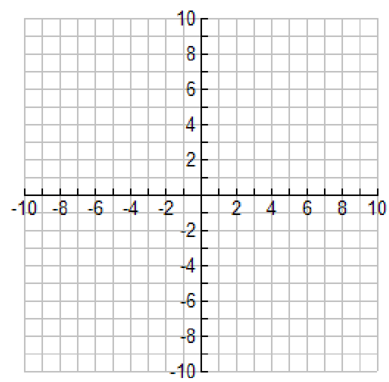
Reflection: y-axis



YOU DO

Translation: $(x, y) \rightarrow (x - 6, y + 2)$

Reflection: x-axis



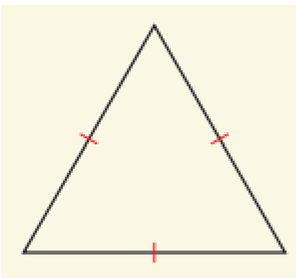
Identifying Lines of Symmetry

A figure in the plane has **line symmetry** when the figure can be mapped onto itself by a reflection in a line. This line of reflection is a **line of symmetry**, such as line *m* at the left. A figure can have more than one line of symmetry.

Examples: Determine the number of lines of symmetry for each figure.

WE DO

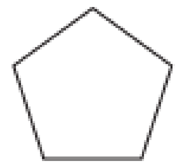
1.)



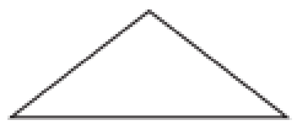
2.)

YOU DO

3.)



4.)



Assignment	

4.3 Be able to perform rotations.____ Date:_____________

Define Vocabulary:

rotation –

center of rotation –

angle of rotation –

rotational symmetry –

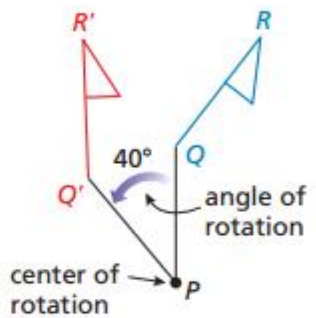
center of symmetry –

Rotations

A **rotation** is a transformation in which a figure is turned about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q' so that one of the following properties is true.

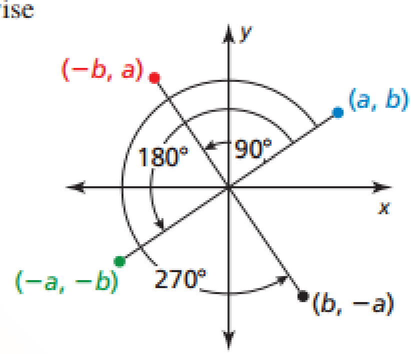
- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then $Q = Q'$.



Coordinate Rules for Rotations about the Origin

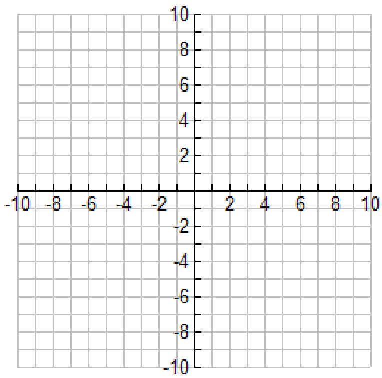
When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90°, $(a, b) \rightarrow (-b, a)$.
- For a rotation of 180°, $(a, b) \rightarrow (-a, -b)$.
- For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.

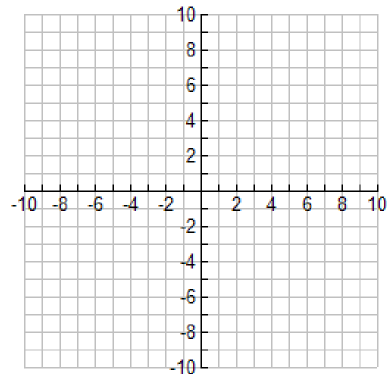


Graph triangle *ABC* with the vertices A(3, 1) and B(3, 4), and C(1, 1) and its image after the described rotation.

180°



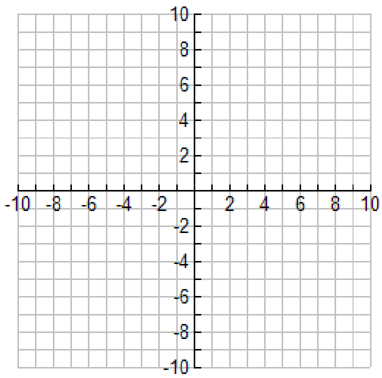
270°



<u>YOU DO</u>

Graph triangle *ABC* with the vertices A(3, 0) and B(4, 3), and C(6, 0) and its image after the described rotation.

90°



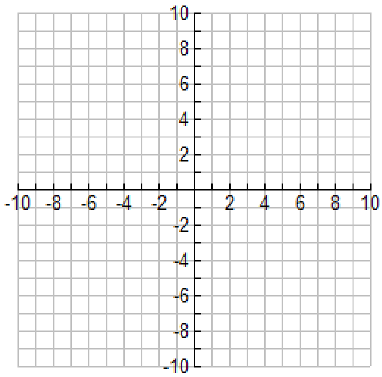
Examples: Rotations

<u>WE DO</u>

Graph $\overline{RS}$ with the vertices R(1, -3) and S(2, -6) and its image after the composition.

Rotation: 180° about the origin

Reflection: y-axis

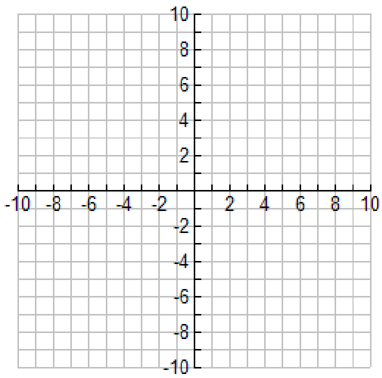


<u>YOU DO</u>

Graph $\overline{RS}$ with the vertices R(-4, 4) and S(-1,7) and its image after the composition.

Translation: $(x, y) \rightarrow (x - 2, y - 1)$

Rotation: 90° about the origin



Assignment	

4.4 Congruence and Transformations__________________________________ Date:________________

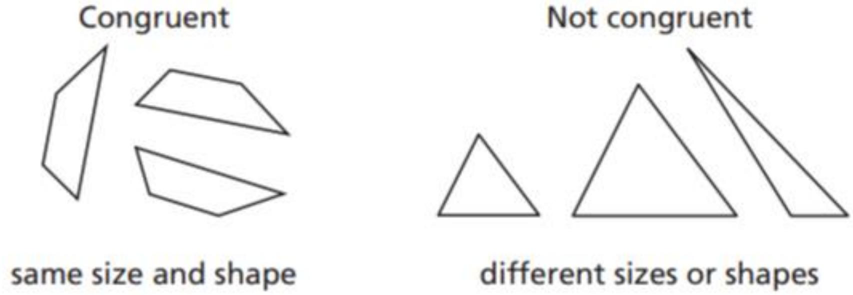
Define Vocabulary:

Congruent Figures –

Congruence transformation –

Identifying Congruent Figures

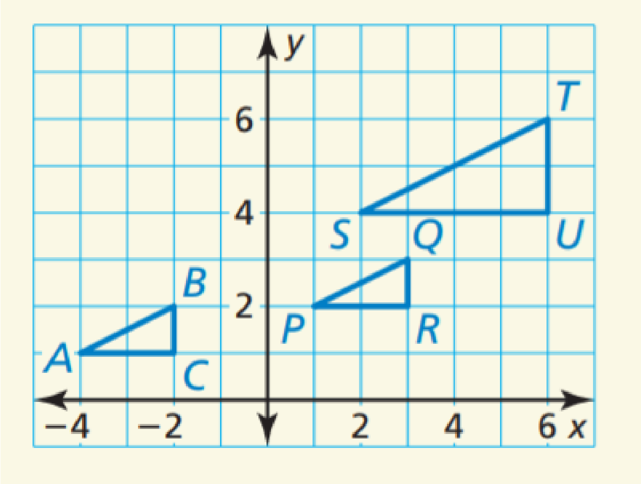
Two geometric figures are **congruent figures** if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other. Congruent figures have the same size and shape.



You can identify congruent figures in the coordinate plane by identifying the rigid motion or composition of rigid motions that maps one of the figures onto the other. Recall from Postulates 4.1–4.3 and Theorem 4.1 that translations, reflections, rotations, and compositions of these transformations are rigid motions.

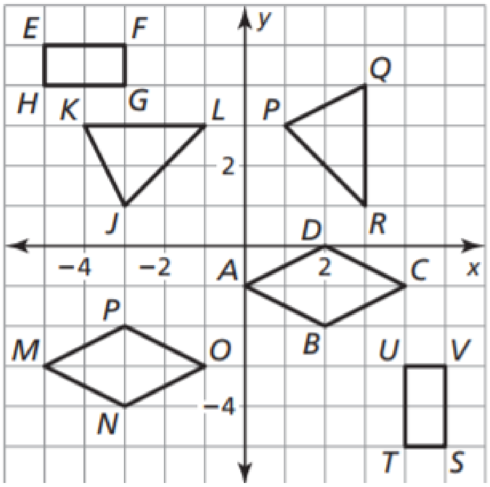
WE DO

Identify any congruent figures in the coordinate plane. Explain.



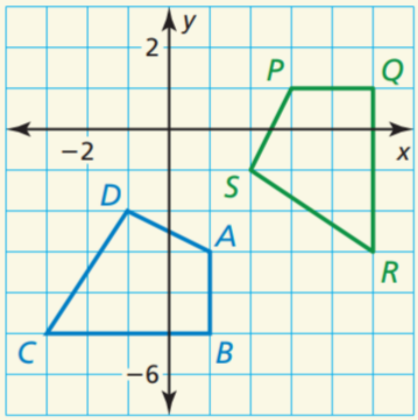
YOU DO

Identify any congruent figures in the coordinate plane. Explain.



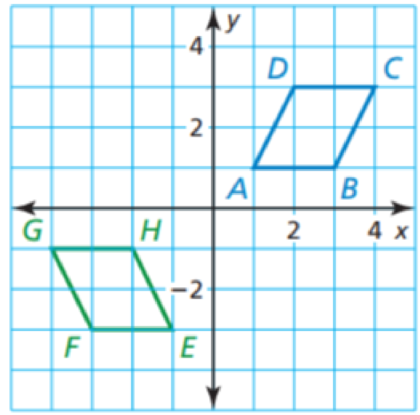
WE DO

Describe a congruence transformation that maps quadrilateral *ABCD* to quadrilateral *PQRS*.

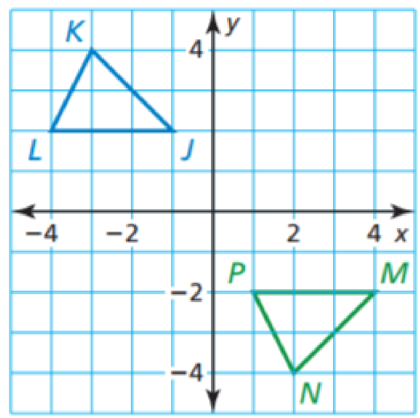


YOU DO

Describe as many congruence transformations that maps ▱ABCD to ▱EFGH as you can find.



Describe a congruence transformation that maps △JKL to △MNP.

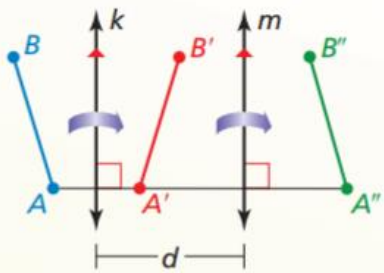


Theorem 4.2 Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If A'' is the image of A, then

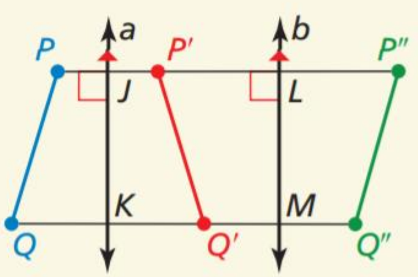
1. $\overline{AA''}$ is perpendicular to k and m, and
2. $AA'' = 2d$, where d is the distance between k and m.



Proof Ex. 31, p. 206

WE DO

In the diagram, a reflection in line a maps $\overline{PQ}$ to $\overline{P'Q'}$. A reflection in line b maps $\overline{P'Q'}$ to $\overline{P''Q''}$. Also, $PJ = 3$ and $LP'' = 8$.



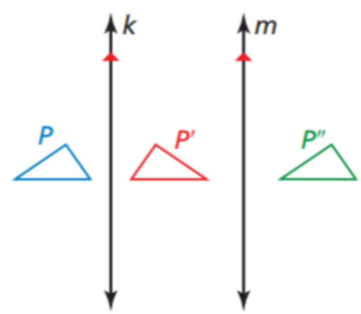
a. Name any segments congruent to each segment: $\overline{PQ}$, $\overline{PJ}$, and $\overline{QK}$.

b. Does $JK = LM$? Explain.

c. What is the length of $\overline{PP''}$?

YOU DO

Use the figure. The distance between line k and line m is 1.6 centimeters.



4. The preimage is reflected in line k, then in line m. Describe a single transformation that maps the blue figure to the green figure.

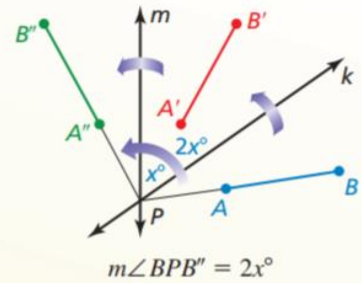
5. What is the relationship between $\overline{PP'}$ and line k? Explain.

6. What is the distance between P and P''?

Theorem 4.3 Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P, then a reflection in line k followed by a reflection in line m is the same as a rotation about point P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by lines k and m.

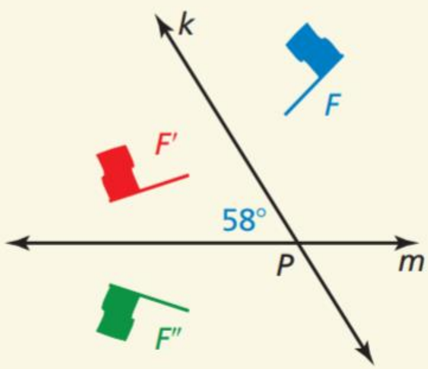


$m\angle BPB'' = 2x°$

Proof Ex. 31, p. 250

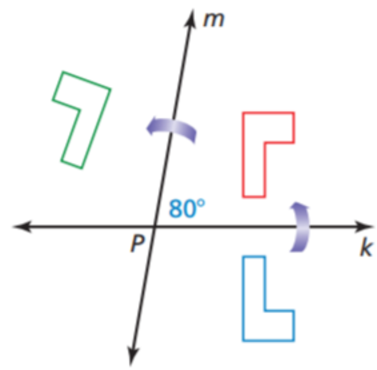
WE DO

In the diagram, the figure is reflected in line k. The image is then reflected in line m. Describe a single transformation that maps F to F''.



YOU DO

7. In the diagram, the preimage is reflected in line k, then in line m. Describe a single transformation that maps the blue figure onto the green figure.



8. A rotation of 76° maps C to C'. To map C to C' using two reflections, what is the measure of the angle formed by the intersecting lines of reflection?

Assignment	

<u>4.5 Be able to perform dilations.</u> ______________________________________ **<u>Date:</u>**____________

Define Vocabulary:

dilation –

center of dilation –

scale factor –

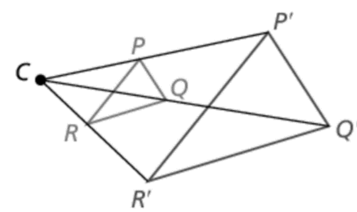
enlargement –

reduction –

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.



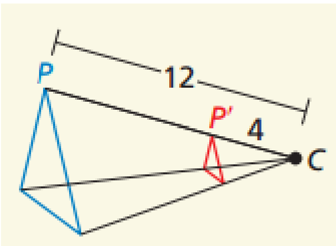
- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \dfrac{CP'}{CP}$.
- Angle measures are preserved.

When the scale factor $k > 1$, a dilation is an **enlargement**. When $0 < k < 1$, a dilation is a **reduction**.

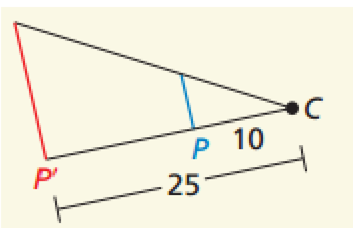
Examples: Find the scale factor of the dilation. Then tell whether the dilation is a reduction or enlargement.

<u>WE DO</u>

1.)

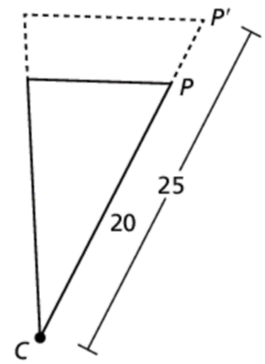


2.)

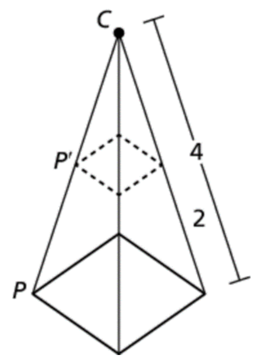


<u>YOU DO</u>

3.)

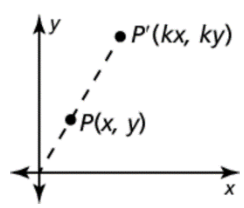


4.)



Coordinate Rule for Dilations

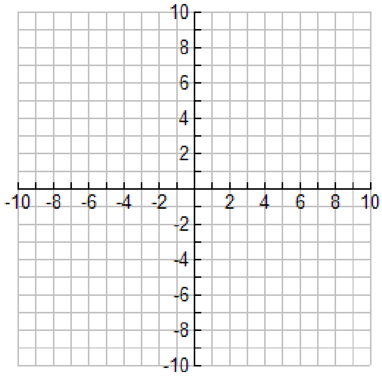
If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.



Examples: Graph the dilation.

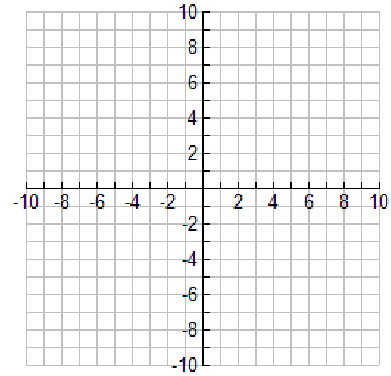
<u>WE DO</u>

Graph ΔPQR with the vertices P(0, 2), Q(1, 0), and R(2, 2) and its image after a dilation of $k = 3$.

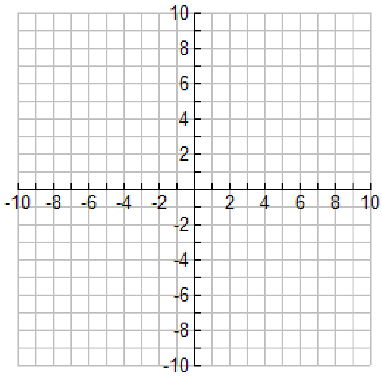


<u>YOU DO</u>

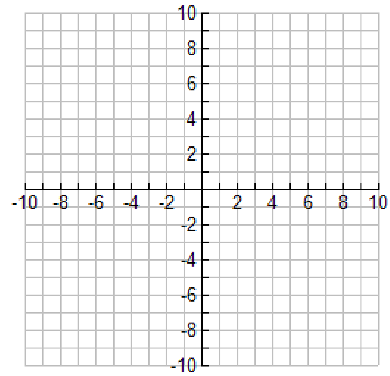
Graph ΔPQR with the vertices P(5, -5), Q(10, -5), and R(10, 5) and its image after the dilation of $k = .4$



Graph ΔPQR with the vertices P(3, 6), Q(3, -3), and R(6, 6) and its image after a dilation of $k = -\frac{1}{3}$.



Graph ΔPQR with the vertices P(1, 2), Q(3, 1), and R(1, -3) and its image after the dilation of $k = -2$



Examples: Finding a scale factor

WE DO

You design a poster on an 8.5-inch by 11-inch paper for a contest at your school. The poster of the winner will be printed on a 34-inch by 44-inch canvas to be displayed. What is the scale factor of this dilation?

A biology book shows the image of an insect that is 10 times its actual size. The image of the insect is 8 centimeters long. What is the actual length of the insect?

YOU DO

You are using word processing software to type the online school newsletter. You change the size of the text in one headline from 0.5 inch tall to 1.25 inches tall. What is the scale factor of this dilation?

You are using a magnifying glass that shows the image of an object that is six times the object's actual size. The image of a spider seen through the magnifying glass is 13.5 centimeters long. Find the actual length of the spider.

Assignment	

<u>4.6 Congruence and Transformations</u> ______________________________ **Date:**________________

Define Vocabulary:

Similarity Transformation -

Similar Figures –

Performing Similarity Transformations

A dilation is a transformation that preserves shape but not size. So, a dilation is a nonrigid motion. A **similarity transformation** is a dilation or a composition of rigid motions and dilations. Two geometric figures are **similar figures** if and only if there is a similarity transformation that maps one of the figures onto the other. Similar figures have the same shape but not necessarily the same size.

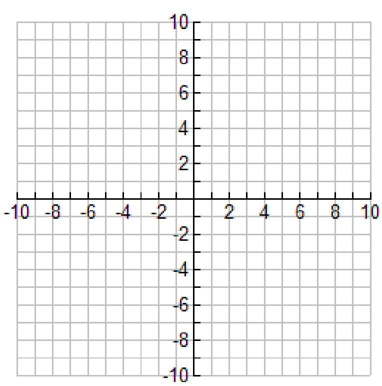
Congruence transformations preserve length and angle measure. When the scale factor of the dilation(s) is not equal to 1 or -1, similarity transformations preserve angle measure only.

<u>WE DO</u>

Graph $\overline{AB}$ with endpoints $A(12, -6)$ and $B(0, -3)$ and its image after the similarity transformation.

Reflection: in the y-axis

Dilation: $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

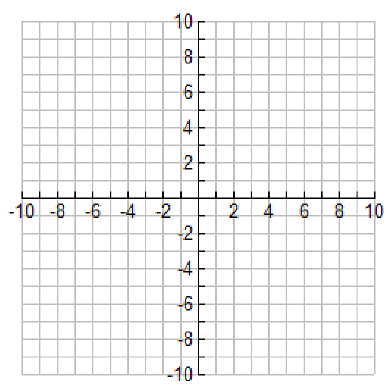


<u>YOU DO</u>

Graph $\triangle FGH$ with vertices $F(1, 2)$, $G(4, 4)$, and $H(2, 0)$ and its image after the similarity transformation.

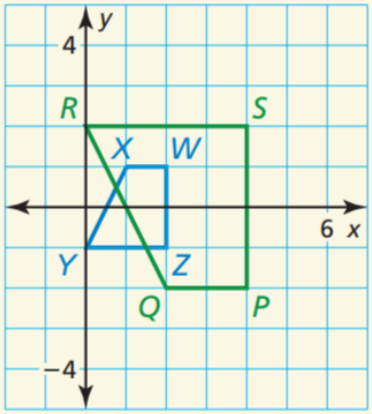
Reflection: in the x-axis

Dilation: $(x, y) \rightarrow (1.5x, 1.5y)$



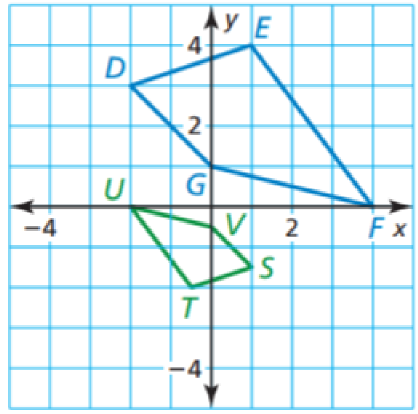
WE DO

Describe a similarity transformation that maps trapezoid *WXYZ* to trapezoid *PQRS*.



YOU DO

Describe a similarity transformation that maps quadrilateral *DEFG* to quadrilateral *STUV*.



Assignment	